Exercise 14

Repeat Exercise 13 for $\mathbf{u} = 3\mathbf{i} + \mathbf{j} - \mathbf{k}$, $\mathbf{v} = -6\mathbf{i} - 2\mathbf{j} - 2\mathbf{k}$.

Solution

Write each of the vectors as

$$\begin{aligned}\mathbf{u} &= (3, 1, -1)\\ \mathbf{v} &= (-6, -2, -2).\end{aligned}$$

Then

$$\begin{aligned}\|\mathbf{u}\| &= \sqrt{3^2 + 1^2 + (-1)^2} = \sqrt{11}\\ \|\mathbf{v}\| &= \sqrt{(-6)^2 + (-2)^2 + (-2)^2} = \sqrt{44} = 2\sqrt{11}\\ \mathbf{u} + \mathbf{v} &= (3, 1, -1) + (-6, -2, -2) = (-3, -1, -3)\\ \mathbf{u} \cdot \mathbf{v} &= (3, 1, -1) \cdot (-6, -2, -2) = (3)(-6) + (1)(-2) + (-1)(-2) = -18\\ \mathbf{u} \times \mathbf{v} &= \begin{vmatrix} \hat{\mathbf{x}} & \hat{\mathbf{y}} & \hat{\mathbf{z}} \\ 3 & 1 & -1 \\ -6 & -2 & -2 \end{vmatrix} = (-2 - 2)\hat{\mathbf{x}} - (-6 - 6)\hat{\mathbf{y}} + (-6 + 6)\hat{\mathbf{z}} = -4\hat{\mathbf{x}} + 12\hat{\mathbf{y}} = (-4, 12, 0).\end{aligned}$$